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256. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\sum\limits_{x=0}^{x=\infty}\frac{x^{2m}}{1+x^{2n}}=\frac{\pi}{2n\sin\frac{2m+1}{2n}\pi}$, m and n being positive integers of which n is the greater.

I. Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$\sum_{x=0}^{x=\infty}\frac{x^{2m}}{1+x^{2n}}=\int_0^\infty\frac{x^{2m}dx}{1+x^{2n}}.$$

Let $x^{2n}=y$, and let $\frac{2m+1}{2n}=r$. Then

$$\int_0^\infty\frac{x^{2m}dx}{1+x^{2n}}=\frac{1}{2n}\int_0^\infty\frac{y^{r-1}dy}{1+y}=\frac{1}{2n}\Gamma(r)\Gamma(1-r)=\frac{\pi}{2n\sin r\pi}=\frac{\pi}{2n\sin\frac{2m+1}{2n}\pi}.$$

II. Solution by V. M. SPUNAR, Mechanical and Civil Engineer, East Pittsburg, Pa.

This problem belongs in a restricted sense to Algebra; as, however, the process is intimately connected with integration, let α_1, α_2, α_3, ..., α_n denote the roots of $x^{2n}+1=0$, the roots being real and unequal; hence, by the theory of equations, α is of the form

$$\alpha=\cos\frac{(2k+1)\pi}{2n}+i\sin\frac{(2k+1)\pi}{2n},$$

in which k is either 0 or a positive integer less than n [$k=0$, 1, ..., $(n-1)$]. Again, by the method of decomposition, we have

$$A-Bi=\frac{\alpha^{2m}}{2n\,\alpha^{2n-1}}=-\frac{\alpha^{2m+1}}{2n}.$$

$$\therefore \alpha^{2m+1}=\cos(2k+1)\phi+i\sin(2k+1)\phi,\text{ where }\phi=\frac{(2m+1)\pi}{2n}.$$

Hence, $B=\frac{\sin(2k+1)\phi}{2n}$.

$$\therefore B_1+B_2+B_3+\ldots+B_n=\frac{1}{2n}[\sin\phi+\sin3\phi+\ldots+\sin(2n-1)\phi]$$

$$=\frac{1}{2n}\left(\frac{1}{\sin\frac{(2m+1)\pi}{2n}}\right),$$ by a well known formula in Trigonometry.

$(B_1+B_2+B_3+\ldots+B_n)2\pi=\int_{-\infty}^{\infty}\frac{x^{2m}dx}{1+x^{2n}}$, by a well known formula in Calculus.

$$\text{Hence, } \sum_{x=-\infty}^{x=+\infty}\frac{x^{2m}}{1+x^{2n}}=\frac{\pi}{n\sin\frac{(2m+1)\pi}{2n}}=2\sum_{x=-0}^{x=+\infty}\frac{x^{2m}}{1+x^{2n}}.$$

$$\text{Hence, } \sum_{x=0}^{x=+\infty}\frac{x^{2m}}{1+x^{2n}}=\frac{\pi}{2n\sin\frac{(2m+1)\pi}{2n}}.$$

Also solved by J. I. Wodo.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

120. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Find the prime numbers p for which $x^2-pxz-px-z+p^2-3=0$ has more than two sets of positive integral solutions x, z, each $<p$.

II. Solution by E. B. ESCOTT, Ann Arbor, Mich.

The solution of this problem which was published in the March number of THE AMERICAN MATHEMETICAL MONTHLY does not seem to answer the question exactly, but rather proposes instead another problem which seems equally difficult, as there is no certainty that the numbers p so found will be prime. Thus, the solution given leads to the following values of p, 74 and 394199, neither of which is prime.

The following method will give an indefinite number of primes p which satisfy the given equation, and leads to formulas such as $p=x^2-x-1$;

$$x=n^2+n-1,\ p=n^3+n^2-2n-1. \tag{1}$$

$$\text{In the equation, } z+1=n=\frac{x^2+p^2-2}{px+1}, \tag{2}$$

p and x occur symmetrically, and n will be unchanged if p and x are interchanged. If the roots of the equation (2), considered as an equation in x, are x_1 and x_2, we have $x_2=p_1n-x_1$, and since, as just noticed, p and x may be interchanged, we may take this value of x_2 for a new p (say p_2) and take the new $x_1=p_1$. n will be unchanged. The new $x_2=p_2n-x_1=p_2n-p_1$. Let this equal p_3, etc. Then we see that in the recurring series

P_n : x_1, p_1, p_1n-x_1, ..., where the scale of relation is $P_{n+2}=nP_{n+1}-P_n$,